Empirical formula for the excitation energies of the first 2^+ and 3^- states in even-even nuclei

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Abstract

We report empirical findings that a simple formula in terms of the mass number A, the valence proton number N_p, and the valence neutron number N_n can describe the essential trends of excitation energies E_x of the first 2^+ and 3^- states in even-even nuclei throughout the periodic table. The formula reads as $E_x = \alpha A^{-\beta} + \exp(-\lambda N_p) + \exp(-\lambda N_n)$. The parameter β in the first term is determined by the mass number A dependence of the bottom contour line of the excitation energy systematics. The other two parameters α and λ are fitted by minimizing the χ^2 value between the measured and calculated excitation energies. Our results suggest that the single large-j shell simulation can be applied to the excitation energies of the first 2^+ and 3^- states in even-even nuclei.

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The exhaustive compilation of the $B(E2)$ values between the 0^+ ground state and the first 2^+ state in even-even nuclei by Raman *et al.* has provided a rare opportunity to make a systematic study of the relevant nuclear properties throughout the periodic table [1, 2]. For example, it was seen that $B(E2)$ values, when plotted in terms of the atomic number Z, showed a mid-shell bump within a major shell consisting of several single particle levels. It has also been shown that this bump could be nicely explained by an idea of the single large-j shell simulation [3, 4]. Within this simulation, the nuclei corresponding to the j_z sub-states of the large angular momentum j are described in terms of the valence nucleon numbers N_p and N_n which are then adopted to parameterize the various quantities such as the $B(E2)$ values and the nuclear quadrupole moments. The valence proton(neutron) number $N_p(N_n)$ is defined as the number of proton(neutron) particles below mid-shell or the number of proton(neutron) holes past mid-shell within the given major shell. Hamamoto, who introduced the concept of the valence nucleon numbers for the first time more than four decades ago, has shown that the square root of the ratio of the measured and the single-particle $B(E2)$ values, $[B(E2)_{\rm exp}/B(E2)_{\rm sp}]^{1/2}$, is roughly proportional to the product N_pN_n [5]. Casten has extended the idea and shown that if one parameterizes the collective variables or operators in terms of the product N_pN_n instead of the usual mass number A, the neutron number N, or the proton number Z, then one gets a substantial reduction in the number of parameters without serious loss of accuracy [6, 7].

In this Letter, we want to illustrate that the valence nucleon numbers can be employed in describing not only the $B(E2)$ values as shown in the previous publications by other authors [5, 6] but also the excitation energies themselves of the first 2^+ states in the even-even nuclei. In addition, it will also be shown that a single formula in terms of the valence nucleon numbers reproduces the essential trends of the first 2^+ excitation energies throughout the periodic table. This fact presents a noticeable contrast to the former studies where the parametrization by the valence nucleon numbers N_p and N_n was performed for nuclei which belong to one major shell only.

Evidence in favor of the single large-j shell structure in the excitation energies $E_x(2_1^+)$ of the first 2^+ states can be seen from Fig. 1(a) which shows the data quoted from Ref. [2] where the best known values of $E_x(2_1^+)$ have been compiled for even-even nuclei. First, consider the solid lines in the upper panel of Fig. 1(a) which connect the excitation energies of the nuclei that belong to the same isotopic chain. Let us focus on the V-shaped portion which can

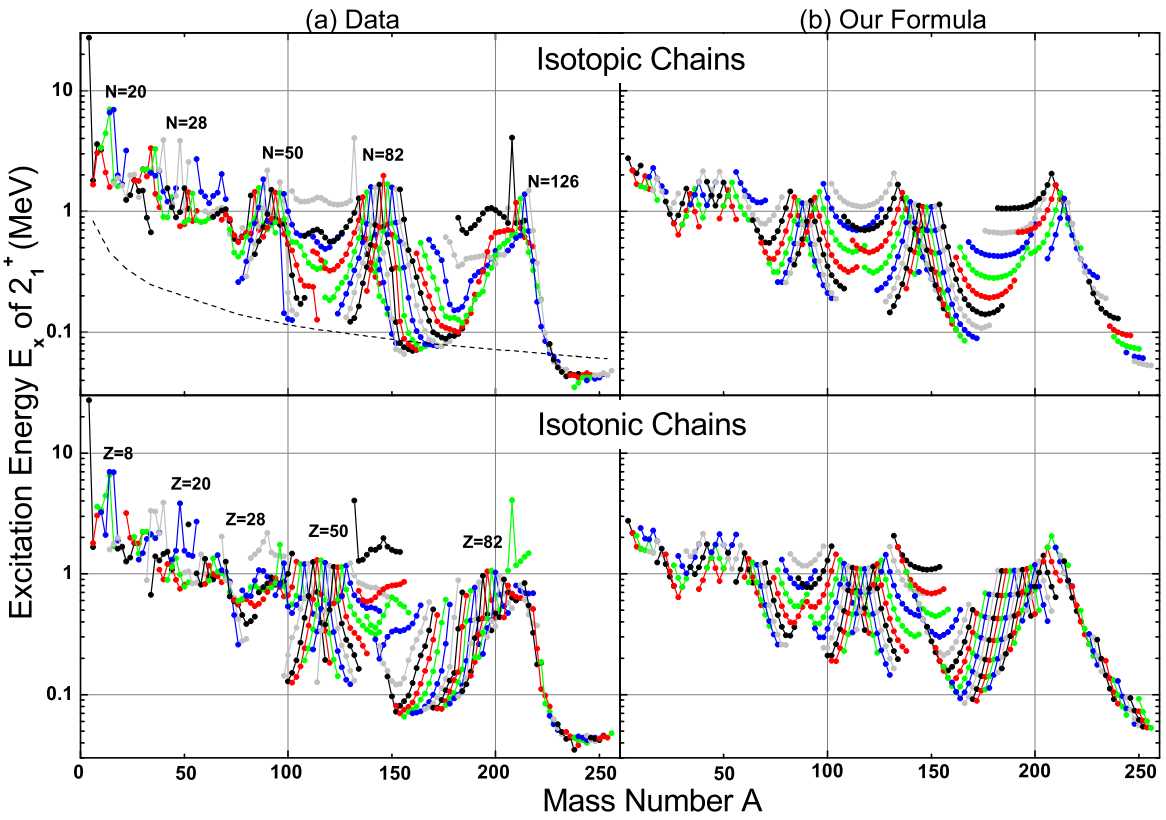

FIG. 1: The excitation energies of the first 2^+ states in even-even nuclei. Solid lines in the upper(lower) panels represent isotopic(isotonic) chains. The left panels denoted by (a) show the data quoted from Ref. [2] and the right panels denoted by (b) show the results obtained by our formula, Eq. (1). (In electronic version, the color code for an isotopic(isotonic) chain in the part (a) is the same as the color code for the corresponding isotopic(isotonic) chain in the part (b).)

be easily identified from each one of isotopic chains in Fig. 1(a). Those nuclei which form a V-shape actually constitute one of the neutron major shells (with the proton number fixed) in the single large-j shell simulation. The V-shape itself tells us that, within a major shell, the excitation energy $E_x(2_1^+)$ is minimum in the mid-shell nucleus which corresponds to the bottom of the V-shape and $E_x(2_1^+)$ becomes larger either when the neutron number increases or decreases from the mid-shell. Furthermore, as indicated by numbers in the upper panel of Fig. 1(a), the neutron number N of the nucleus at the top of V-shapes belongs to one of magic numbers 8, 20, 28, 50, 82, and 126 which form the boundary between adjacent major shells within the model of the single large-j shell simulation [4]. Now, turn to the lower panel of Fig. 1(a). The data points in the lower panel are exactly the same with those in the

upper panel, but the solid lines in the lower panel of Fig. 1(a) are obtained by connecting the excitation energies of the nuclei that belong to the same isotonic chain with different proton numbers. From the isotonic chains in the lower panel of Fig. 1(a), we can observe precisely the same kind of evidence for the single large-j proton shell structure (with the neutron number fixed) as for the single large-j neutron shell structure in the upper panel of Fig. 1(a).

As an effort to describe the systematic behavior of $E_x(2_1^+)$ by a formula which is as simple as possible, we try the following equation

$$E_x = \alpha A^{-\beta} + \exp(-\lambda N_p) + \exp(-\lambda N_n) \tag{1}$$

where α, β, and λ are our free parameters to be fitted from the data. The first term of Eq. (1) represents the overall dependence of the excitation energy on the mass number A and the last two terms account for variations of excitation energies which form the V-shape within a major shell in Fig. 1. We divide our fitting procedure into two steps. First, we construct the bottom contour line by connecting the lowest points among the adjacent V's in the upper panel of Fig. 1(a) and fit it by only the first term, $\alpha A^{-\beta}$, of Eq. (1) to determine two parameters α and β. However, we take only the value of β from the first step, and let both α and λ subject to variation in minimizing the χ^2 value given by

$$\chi^2 = \frac{1}{N_0} \sum_{i=1}^{N_0} \left[E_x^{\rm cal}(i) - E_x^{\rm exp}(i) \right]^2 \tag{2}$$

between all the measured excitation energies $E_x^{\rm exp}$ in Fig. 1(a) and the calculated ones $E_x^{\rm cal}$ by Eq. (1) where N_0 is the number of total data points considered. The adopted values of α, β, and λ in Eq. (1) for excitation energies of the first 2^+ states are listed in the first row of Table I. Our adopted bottom contour line, $E_x = \alpha A^{-\beta}$, is shown as the dashed curve in the upper panel of Fig. 1(a).

In Fig. 1(b), the calculated excitation energies of the first 2^+ states in even-even nuclei with our formula, Eq. (1), using the parameter set given in Table I are plotted. The upper and lower panels of Fig. 1(b) show the same results but the curves in the upper panel connect isotopic chains while those in the lower panel connect isotonic chains. One can immediately find a very close resemblance between the curves in Fig. 1(a) and (b), the data and our results, respectively. (In electronic version, the color code for an isotopic(isotonic) chain in Fig. 1(a) is the same as the color code for the corresponding isotopic(isotonic) chain in Fig. 1(b).) It

is quite remarkable to note that a simple formula such as Eq. (1) can reproduce the data not only qualitatively but also quantitatively to some extent. Especially the characteristic shape of the single large-j neutron shell structure in isotopic chains as well as that of the single large-j proton shell structure in isotonic chains is nicely reproduced by our formula.

The role played by the two exponential terms of Eq. (1) in reproducing the data can be seen in Fig. 2 where the excitation energies $E_x(2_1^+)$ for nuclei between $140 \leq A \leq 200$ are displayed. The solid lines of the left part (a) of Fig. 2 are obtained by connecting the excitation energies which belong to isotopic chains while those of the right part (b) are obtained by connecting them which belong to isotonic chains. Let us compare the top two panels showing the data first with the central two panels and then with the bottom two panels. The solid lines of the left central panel are drawn by taking only the first two terms of Eq. (1) which show $\alpha A^{-\beta} + \exp(-\lambda N_n)$, and those of the right central panel are drawn by the similar two terms, $\alpha A^{-\beta} + \exp(-\lambda N_p)$. We can observe that while the lines at the left(right) side of the left(right) central panel explain the data shown in the corresponding top panel quite reasonably, the same lines at the right(left) side of the left(right) central panel can not reproduce the main feature of the data which show the monotonic increase of the excitation energy as the mass number A increased(decreased). Now, by comparing the top two panels with the corresponding bottom ones which are obtained by adding the term $\exp(-\lambda N_p)$ in the case of the left central panel or the term $\exp(-\lambda N_n)$ in the case of the right central panel, we can find that the overall main features of the data are fairly well reproduced.

One can observe from Fig. 1 that the measured excitation energies $E_x(2_1^+)$ for nuclei roughly between $100 \leq A \leq 200$ are explained best by our formula given by Eq. (1) even quantitatively to some extent. In order to examine them in more detail, we extract the

TABLE I: The adopted values of three parameters in Eq. (1) for excitation energies of the two multipole states 2_1^+ and 3_1^-.

Multipole	α	β	λ
2_1^+	2.91	0.70	0.26
3_1^-	32.69	0.67	0.20

corresponding part from Fig. 1 and out of it we create Fig. 3 where the calculated excitation energies $E_x(2_1^+)$ and the data are plotted in the same panel. In Fig. 3(a), the calculated excitation energies of isotopic chains are represented by thick solid curves while the measured ones of corresponding isotopic chains are depicted by solid circles connected by thin solid lines. Fig. 3(b) are drawn from exactly the same data points used in Fig. 3(a) but they are connected for the isotonic chains. However, three quarters of the data points which appear in Fig. 1 are excluded in making Fig. 3 to avoid too much complexity within a graph. It is noticeable in Fig. 3 that our formula, which is able to account for the essential trends in the systematic behavior of the excitation energies $E_x(2_1^+)$ observed in Fig. 1 very successfully, has its shortcomings in explaining either the detailed shape or actual height of the measured isotopic or isotonic chains.

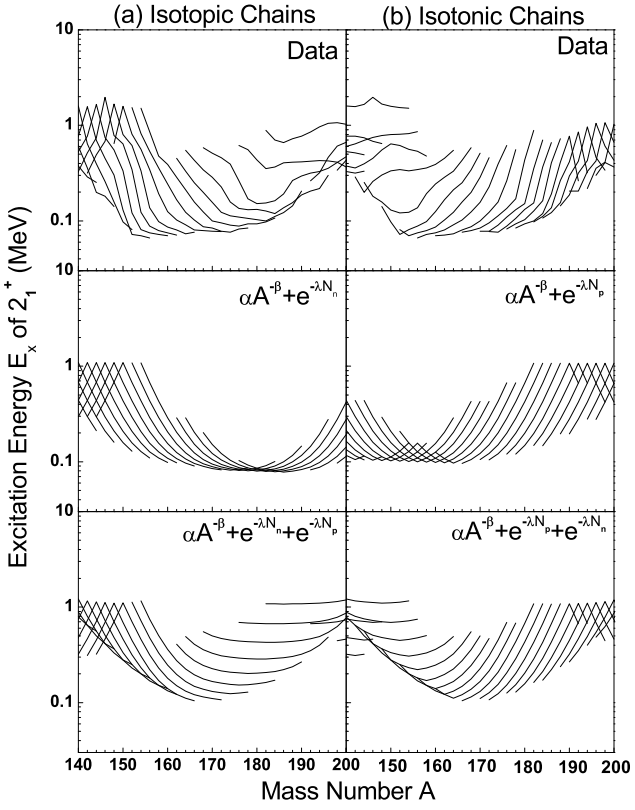

FIG. 2: The excitation energies $E_x(2_1^+)$ for nuclei between $140 \leq A \leq 200$. The solid lines in (a) and (b) represent the isotopic and isotonic chains, respectively. The top, central, and bottom panels show the data, the excitation energies calculated with only the first two terms of Eq. (1), and those calculated with all the three terms of Eq. (1), respectively.

Encouraged by uncovering the fact that the valence nucleon numbers can be employed successfully in explaining the excitation energies of the first 2^+ states in even-even nuclei throughout the periodic table, we decide to apply the above same procedure to the first 3^- states in even-even nuclei. Fortunately, the recently updated compilation of the excitation energies $E_x(3_1^-)$ and the $B(E3)$ values between the 0^+ ground state and the first 3^- state has been already published by Kibédi and Spear [8].

The results for $E_x(3_1^-)$ are presented in Fig. 4. The measured excitation energies shown in Fig. 4(a) are quoted from Ref. [8] and the calculated ones plotted in Fig. 4(b) are obtained by the same formula, Eq. (1). The solid lines connecting data points in Fig. 4 (a) and (b) represent isotopic chains. Additional data points which do not have the measured counterparts in the part (a) are included in the part(b) for the illustrative purpose. The three parameters α, β, and λ in Eq. (1) for $E_x(3_1^-)$ are determined by exactly the same fitting procedure as for $E_x(2_1^+)$ and the final adopted values are listed in the second row of Table I. It is interesting to note that out of three parameters α, β, and λ in Eq. (1), only the value of α for $E_x(3_1^-)$ differs with that for $E_x(2_1^+)$ by a considerable amount, and the values of β and λ for $E_x(3_1^-)$ remain almost the same as them for $E_x(2_1^+)$. One can find a very close resemblance between the data (a) and our results (b) in Fig. 4, and conclude that the characteristic shape of the single large-j shell structure is present in the excitation energies

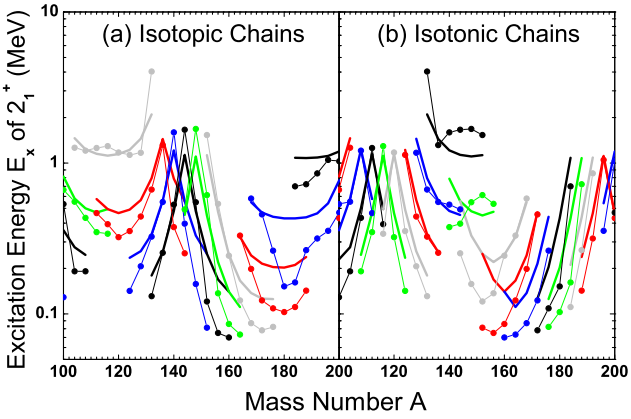

FIG. 3: The measured as well as calculated excitation energies $E_x(2_1^+)$ for nuclei between $100 \leq A \leq 200$. The solid lines in (a) and (b) represent isotopic and isotonic chains, respectively. The calculated excitation energies are represented by thick solid curves while the measured ones are depicted by solid circles and connected by thin solid lines.

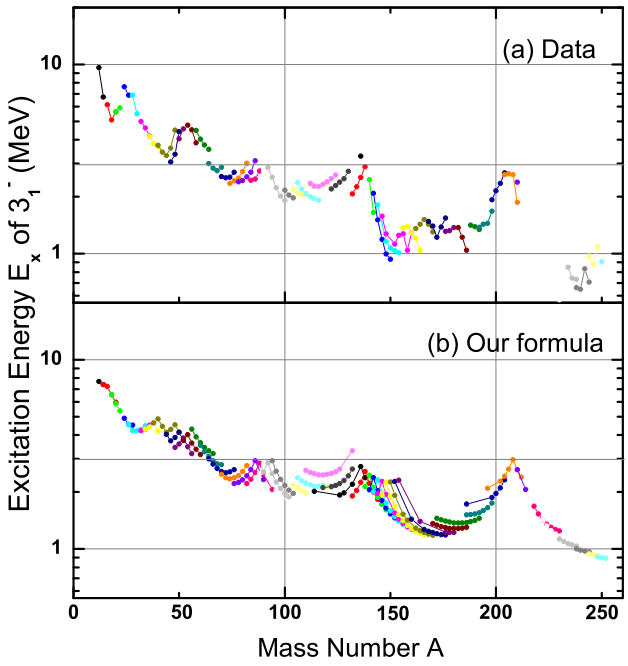

FIG. 4: The excitation energies of the first 3^- states in even-even nuclei. Solid lines connecting the data points represent isotopic chains. The part (a) show the data quoted from Ref. [8] and the part (b) show the results obtained by our formula, Eq. (1).

of the first 3^- states as well as of the first 2^+ states in even-even nuclei.

In summary, we have presented a simple formula which can describe the systematic behavior of excitation energies of the first 2^+ and 3^- states in even-even nuclei. Our formula given by Eq. (1) is composed of only three terms which depend on the mass number A, the valence proton number N_p, and the valence neutron number N_n, respectively. The first term of Eq. (1) represents the overall dependence of the excitation energy on the mass number A while the second(third) term reflects the single large-j proton(neutron) shell structure found in each isotonic(isotopic) chain. We find that Eq. (1) can reproduce the essential trends of the measured excitation energies of the first 2^+ and 3^- states which were compiled extensively in Ref. [2] and Ref. [8], respectively. Furthermore, a preliminary study indicates that Eq. (1) can still be applied to the first 4^+ excitation energies and also to the second 2^+ excitation energies in even-even nuclei [9]. Therefore, it would be most interesting if one can clarify the physics underlying the above observations.

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